Rainfall nowcasting using Burgers' equation

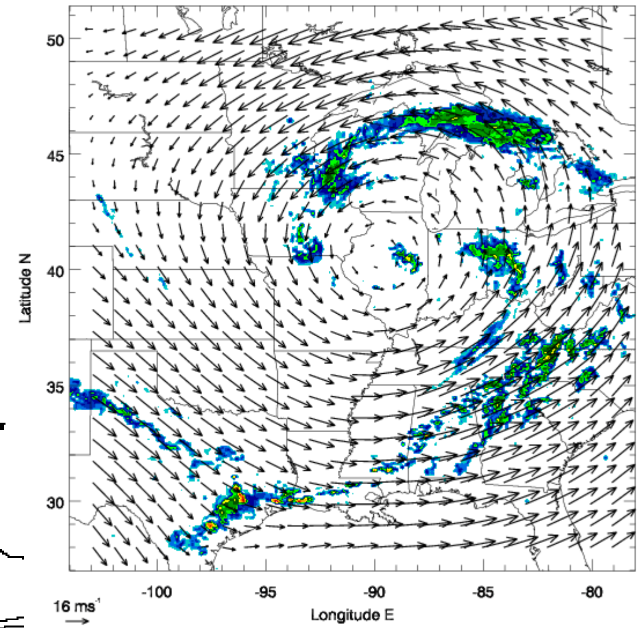
GyuWon Lee, Soorok Ryu

Kyungpook National University,
Daegu, Korea(ROK)

Radar-based nowcasting

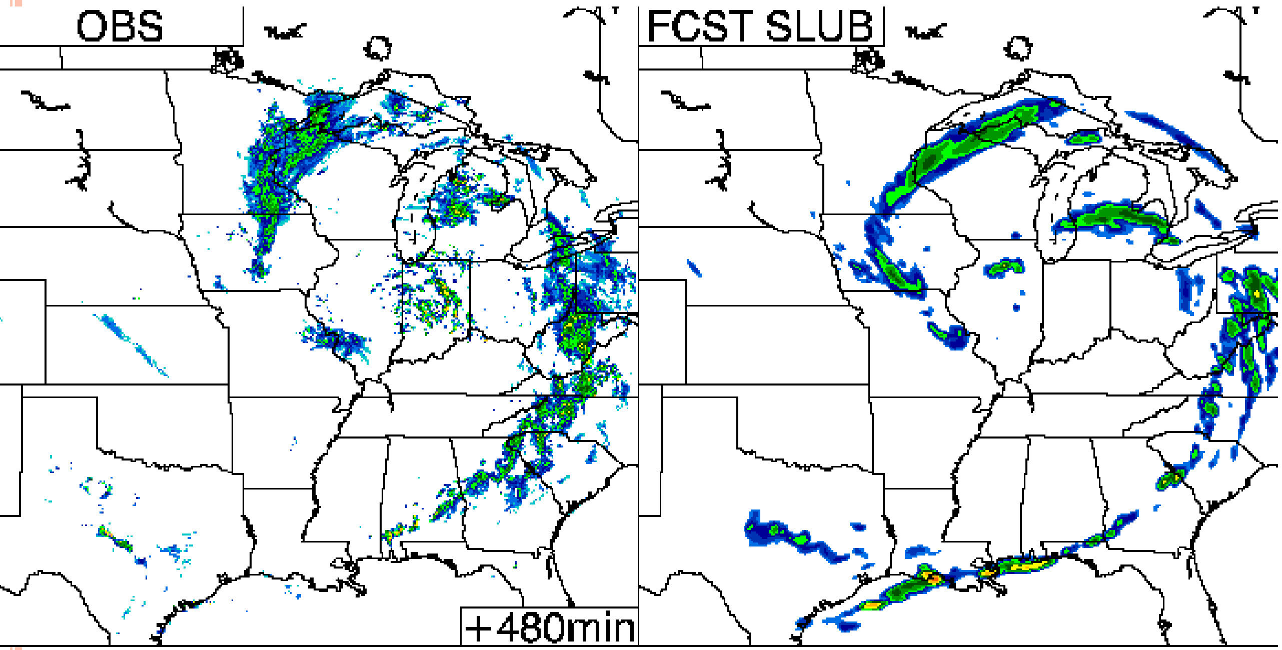
Ex) MAPLE

1. Motion fields of precip. (Variational Echo Tracking: VET)



2. Advect precip. fields: Semi-Lagrangian backward

3. Verification (compare fcst w/ obs)



$$\hat{R}(t_0+\tau,\vec{x}) = R(t_0,\vec{x}-\alpha)$$

Predicted field

Observed field

- Growth/decay (scale of predictability)
- Non-stationary motion fields

Germann and Zawadzki (2002)

METHODOLOGY

Lagrangian extrapolation (advection) $\hat{R}(t_0 + \tau, \vec{x}) = R(t_0, \vec{x} - \alpha)$

OR

Conservation equation

$$\frac{dR}{dt} = \frac{\partial R}{\partial t} + u\frac{\partial R}{\partial x} + v\frac{\partial R}{\partial y} = 0$$

We solved this simple advection equation(AE) directly

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y}$$

: Type 1

Add diffusion term for spatial filtering (smoothing): advection diffusion equation (ADE)

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y} + v\left(\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2}\right)$$

: Type 2

METHODOLOGY

However, above two equations assume that the motion vector field is stationary in time (constant motion vectors for entire forecast time)

Introduce Burgers' equation:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + s\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + s\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

to allow non-stationarity of motion vectors. The s controls the degree of the smoothness.

Methodology

Semi-Lagrangian extrapolation (S-L): $\hat{R}(t_0+\tau,\vec{x}) = R(t_0,\vec{x}-\alpha)$

Type 1: advection equation(AE)

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y}$$

Type 2: advection diffusion equation(ADE)

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y} + v\left(\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2}\right)$$

Type 3: advection equation(AE) + Burgers' equation

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y} \quad + \quad \begin{array}{l} \frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + s(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) \\ \frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + s(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) \end{array}$$

Type 4: advection diffusion equation(ADE) + Burgers' equation

$$\frac{\partial R}{\partial t} = -u\frac{\partial R}{\partial x} - v\frac{\partial R}{\partial y} + v\left(\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2}\right) \quad + \quad \begin{array}{l} \frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + s(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) \\ \frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} + s(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) \end{array}$$

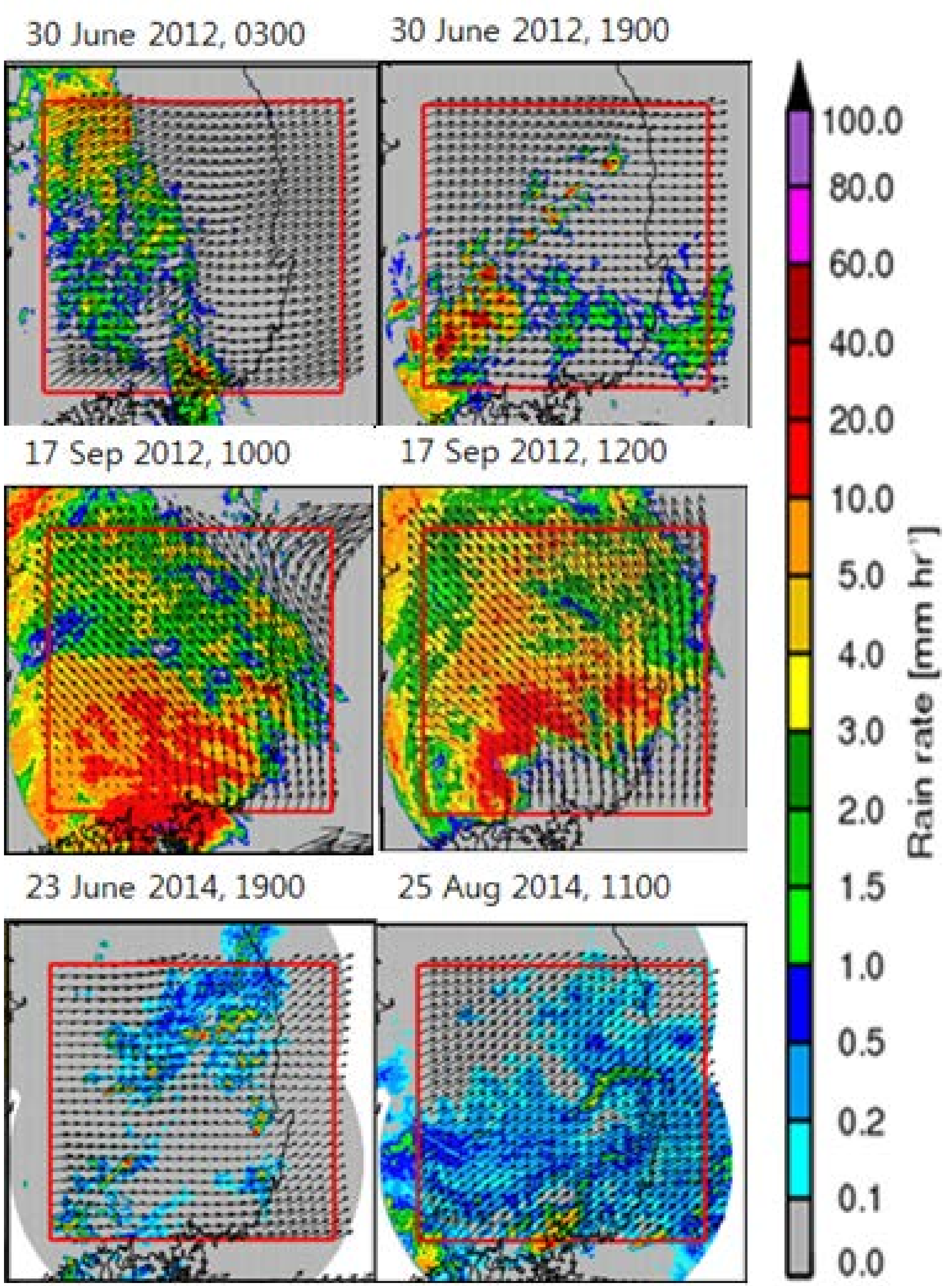
METHODOLOGY

1. **Motion vectors (u, v):** derived from variational echo tracking (VET) with Δt = 2.5 min

2. **Advection (diffusion) equation:**
   - ODE for time: Explicit (forward) Runge-Kutta fourth order (RK4) with Δt = 0.1 min
   - Spatial derivative for R: Finite difference method with Δx=Δy=0.25 km
   - 0.0 $\leq \nu \leq$ 0.1

3. **Burgers' equation:**
   - Spatial derivative for (u,v): Finite difference method with Δx=Δy=10 km
   - s=0.2

DATA



1. Cases

- 6 events, 2.5 min CAPPI composite from 3 radars
- 15 min nowcasting up to 3h

2. Computation domain

- Southeast area in South Korea
- 312 km x 312 km at 0.25 km resolution (1248 x 1248 pixels)
- Motion vectors : 10 km resolution
- Verification domain: 250 km x 250 km (red box)

Skill scores, error statistics

- 2D Contingency table

	Forecast	
Obs	**$R>=R_{th}$**	**$R<R_{th}$**
$R>=R_{th}$	**Hit (a)**	**Miss (c)**
$R< R_{th}$	**False alarm (b)**	**Correct negative (d)**

- Categorical scores

Verification score	Formula
Probability of detection **(POD)**	a/(a+c)
False alarm ratio **(FAR)**	b/(a+b)
Critical success index **(CSI)**	a/(a+b+c)
Equitable threat score **(ETS)**	(a-w)/(a+b+c-w), w=(a+b)(a+c)/(a+b+c+d)

- **Correlation coefficient** :

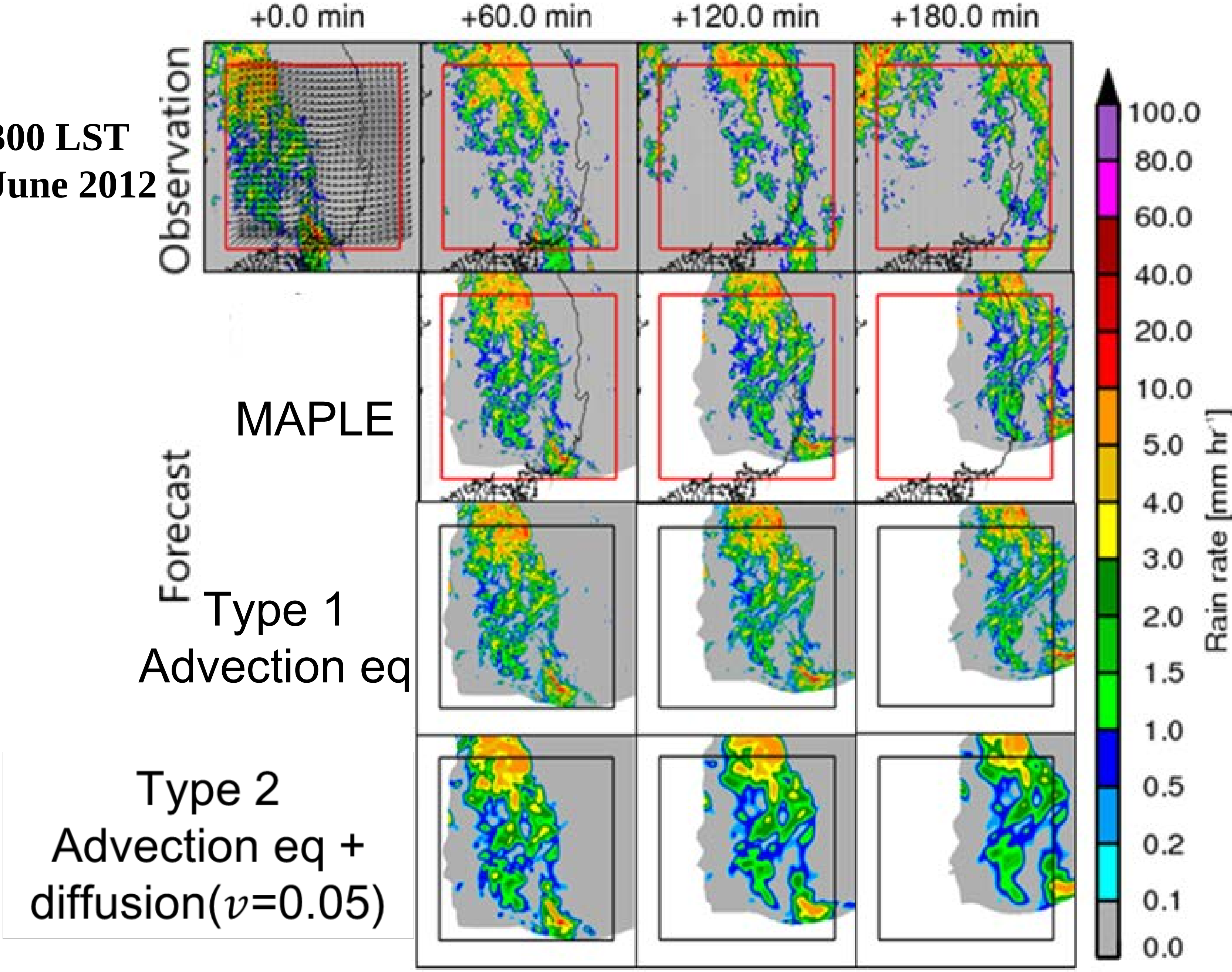
$$r(t) = \frac{\sum_{n(t)}(R_0(t)R_F(t))}{\left(\sum_{n(t)}(R_0)^2\sum_{n(t)}(R_F)^2\right)^{1/2}},$$

- **Mean Absolute Error :** $\mathrm{MAE}(t) = \frac{1}{n(t)}\sum_{n(t)}|R_F(t) - R_O(t)|$

- **Conditional Mean Absolute Error** : $\mathrm{CMAE}(t) = \frac{1}{a(t)}\sum_{a(t)}|R_F(t) - R_O(t)|.$

MAPLE vs. AE + Diffusion

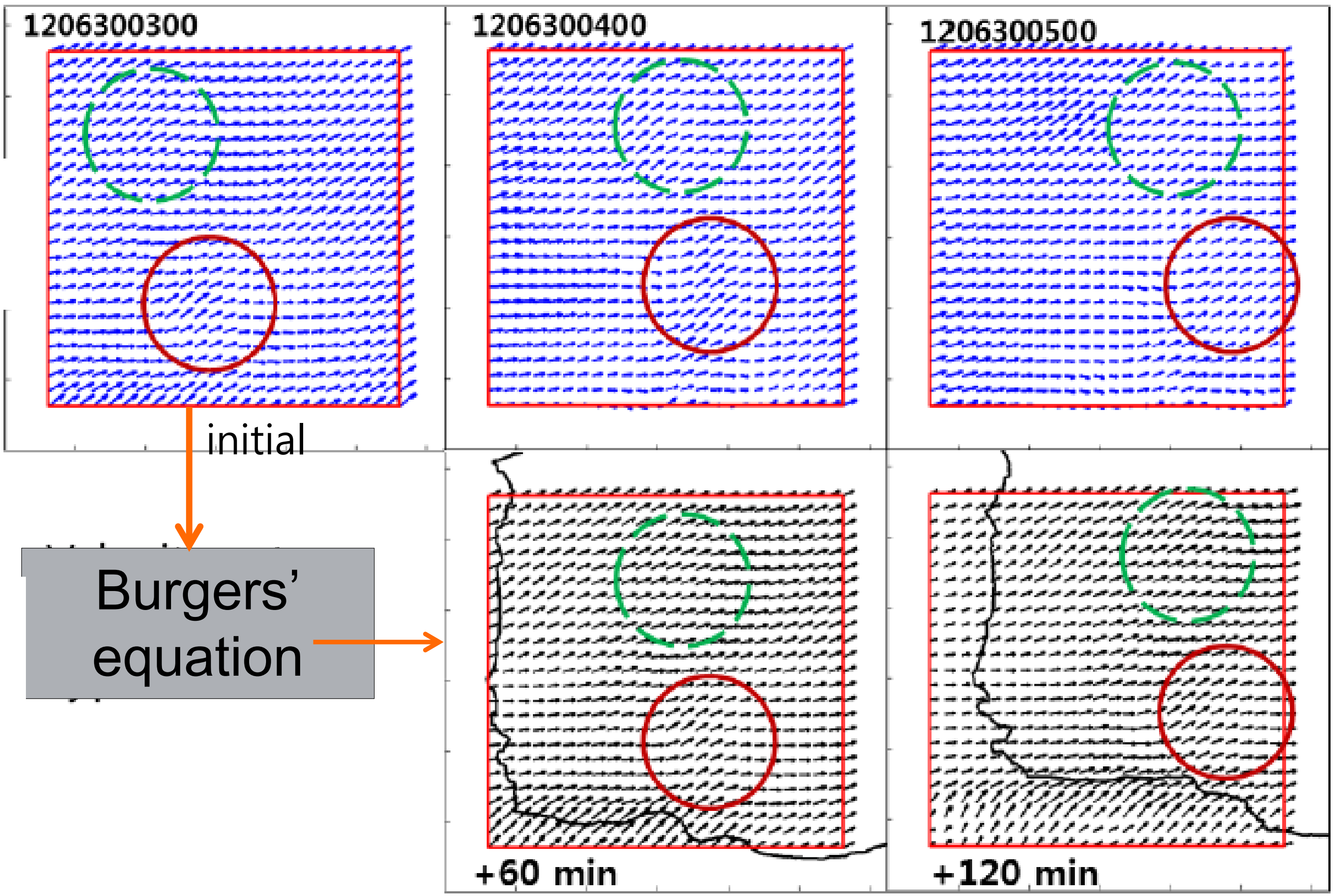
0300 LST
30 June 2012



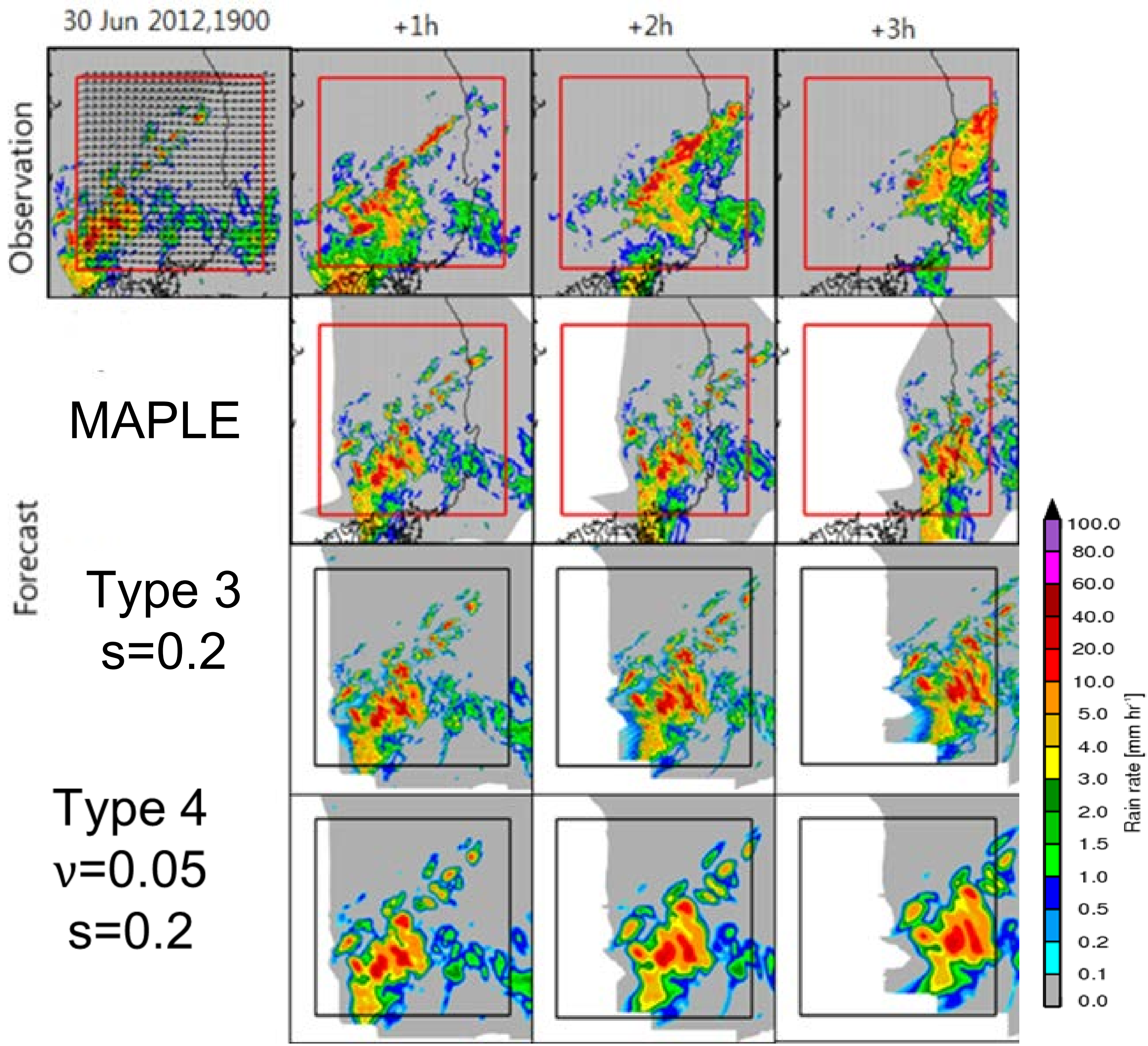
NON-STATIONARY MOTION VECTORS

VET
w/ OBS

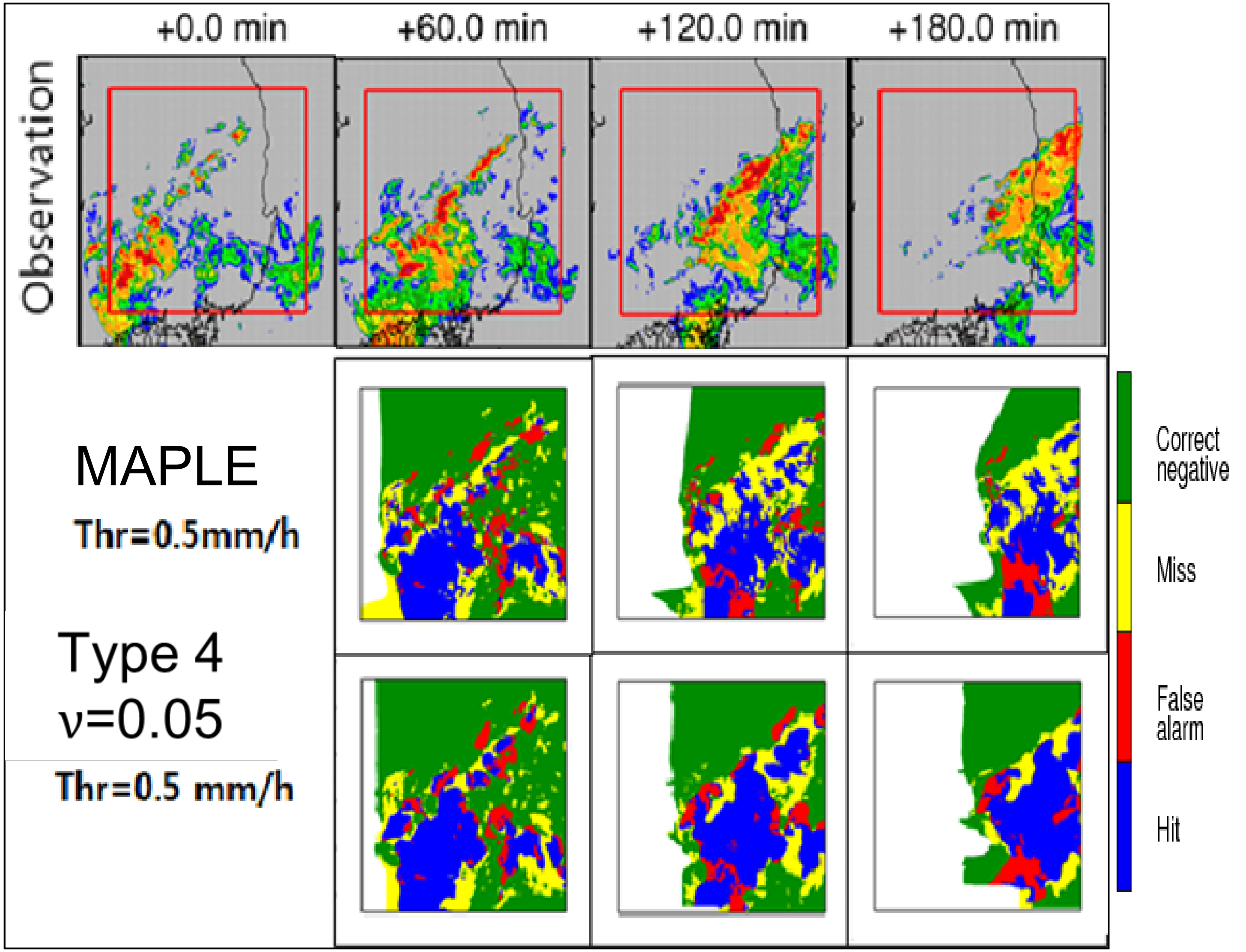
Vectors
w/
Burgers'
eq.



MAPLE vs. AE + non-stationary +Diffusion

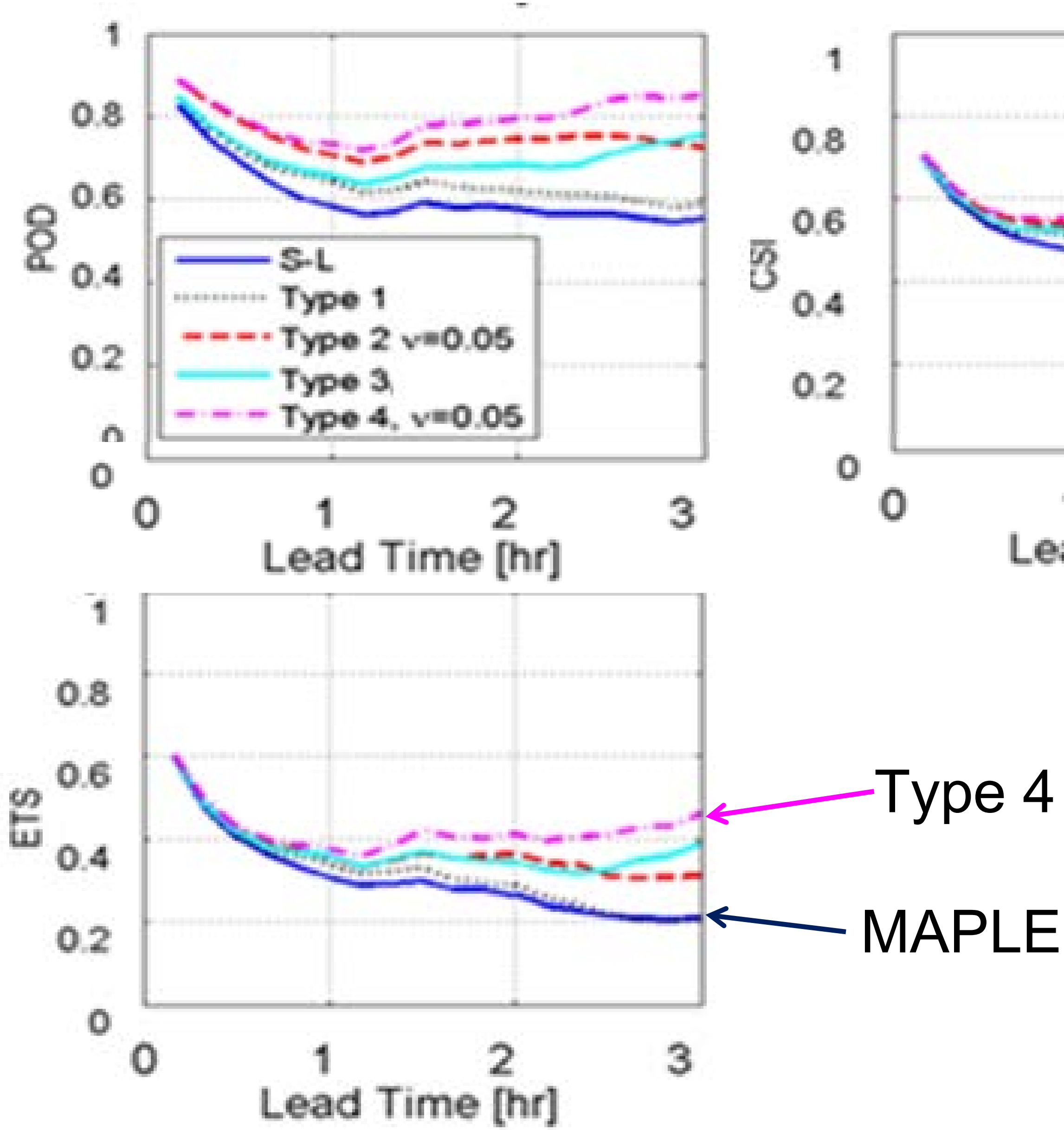


MAPLE VS. AE + NON-STATIONARY + DIFFUSION



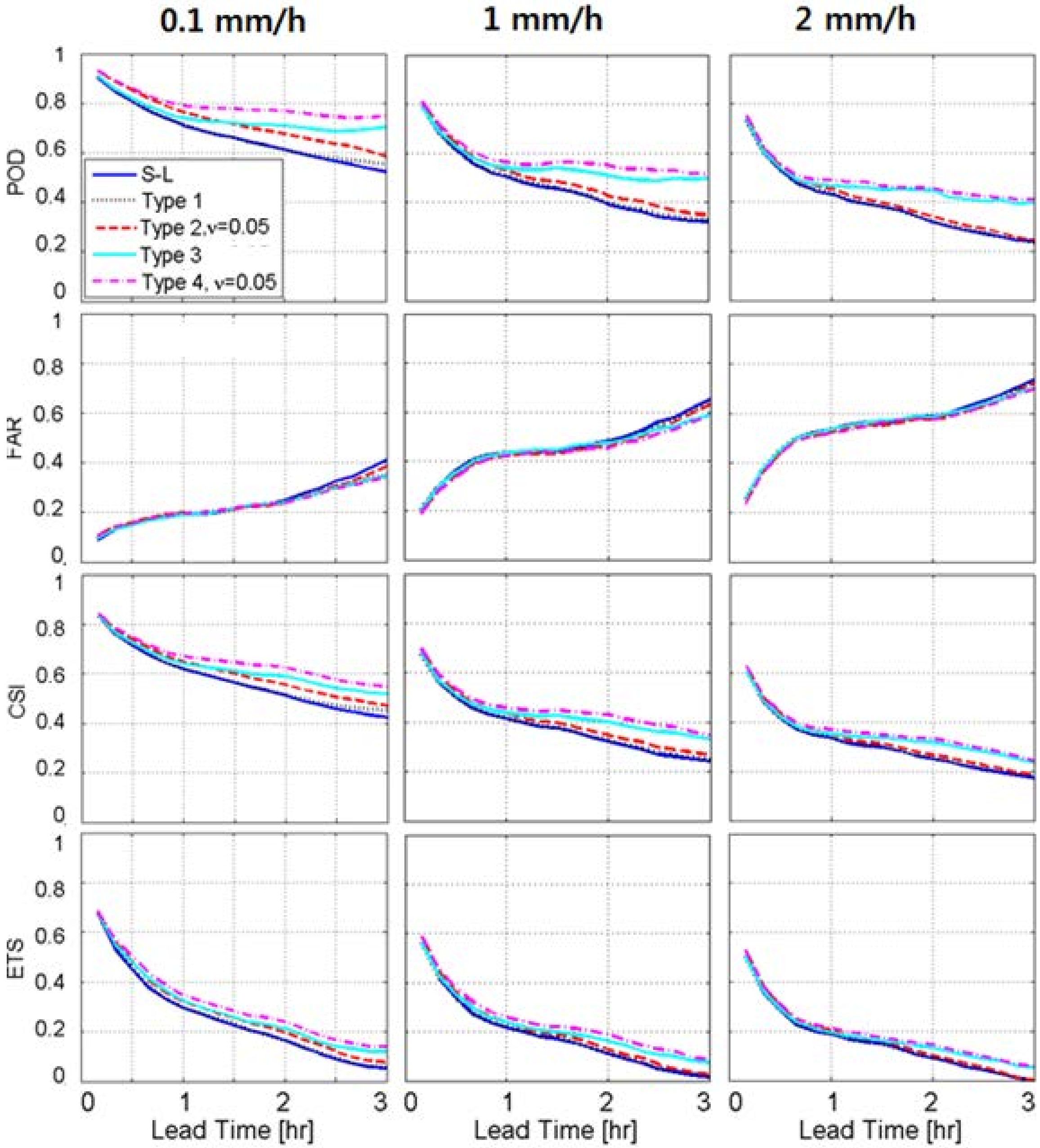
MAPLE vs. AE + non-stationary +Diffusion

Skill scores R_{th} = 0.1mm/h



MAPLE VS. AE + NON-STATIONARY + DIFFUSION

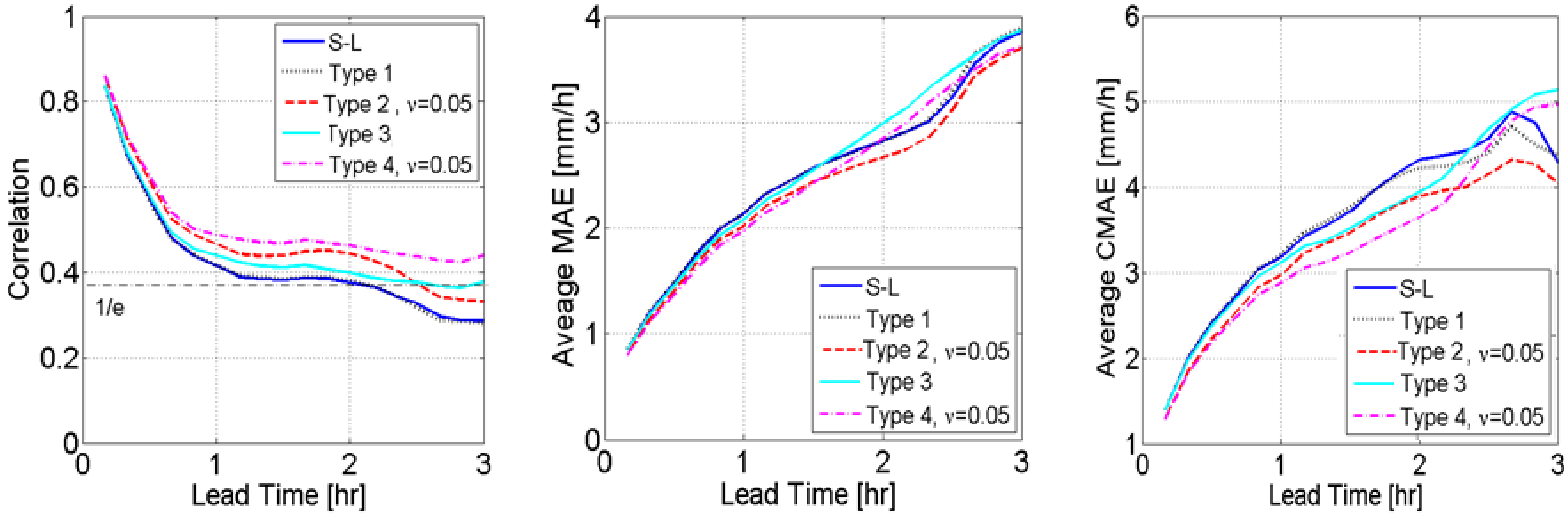
Average skill scores: 6 events



"Type 3, 4"

MAPLE vs. AE + non-stationary + Diffusion

Average skill scores: 6 events



- Lifetime **: Type 4(>3h) > Type 3 (around 3h) > Type 2 (2.5h) > MAPLE (2h) = Type 1**

MAPLE vs. AE + non-stationary + Diffusion

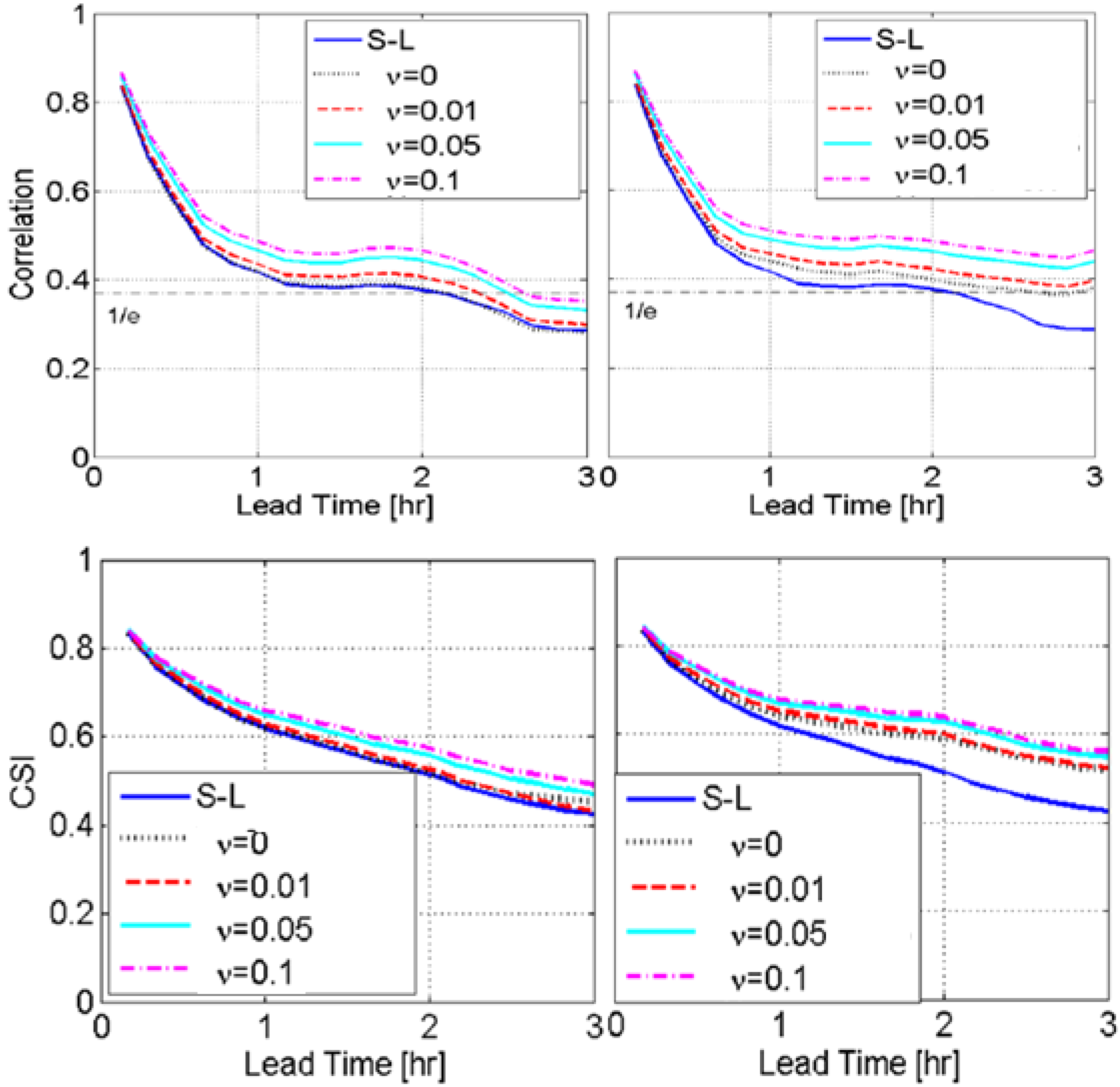
Sensitivity to diffusion

Type 2: diffusion

Type 4: diffusion+Burgers' eq.

Average correlation

Average CSI



NON-STATIONARY MOTION VECTORS

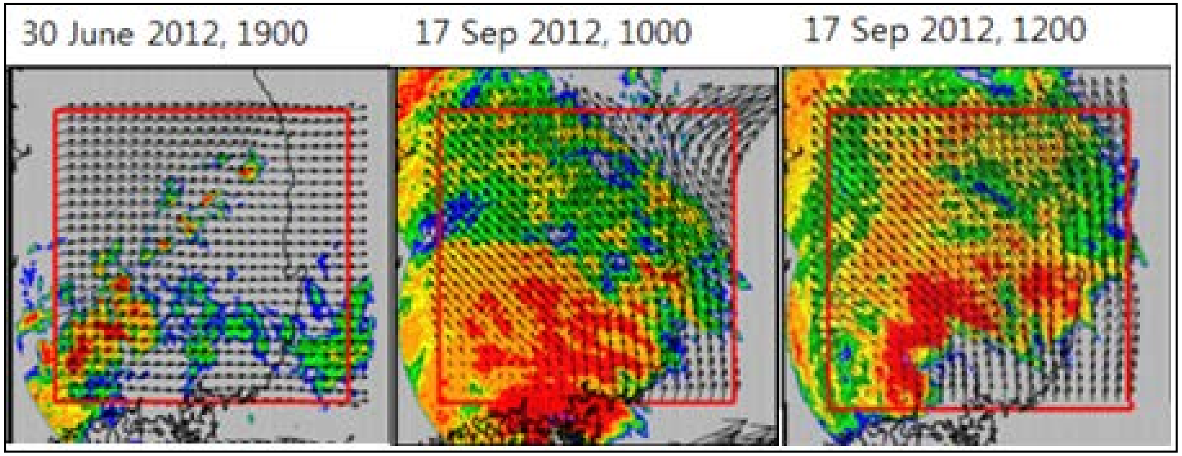
Matrix norm $\|\nabla \cdot \boldsymbol{u}\|$, u(x,y,0) is obtained from VET and u(x,y,t) is from 2D Burgers' equations

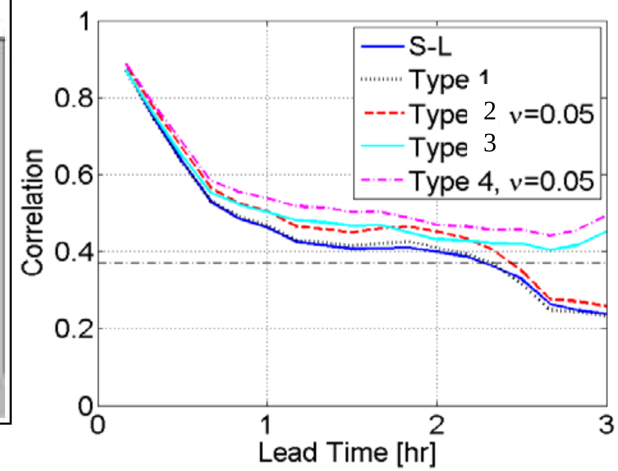
Start Lead time	30 Jun 2012, 0300	30 Jun 2012, 1900	17 Sep 2012, 1000	17 Sep 2012, 1200	23 Jun 2014, 1900	25 Aug 2014, 1100
+0 h	2.01	1.07	5.13	3.08	0.92	1.22
+1 h	2.55	1.05	3.20	2.41	0.89	1.24
+2 h	2.86	1.38	3.12	1.98	0.86	1.18
+3 h	2.35	1.87	2.54	2.25	0.85	1.18
Variance	0.13	0.15	1.27	0.22	0.00	0.00

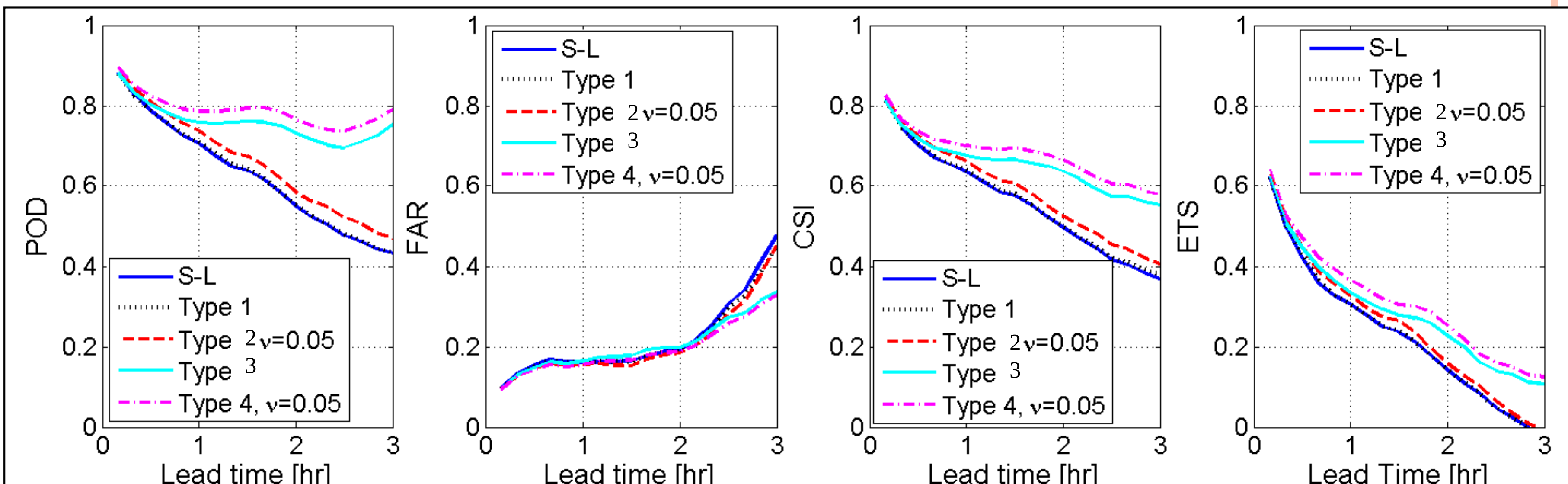
$\mathrm{Var}(\|\nabla \cdot \boldsymbol{u}\|) > 0.13$

NON-STATIONARY MOTION VECTORS

$\mathrm{Var}(\|\nabla \cdot \boldsymbol{u}\|) > 0.13$ ← Non-stationarity?



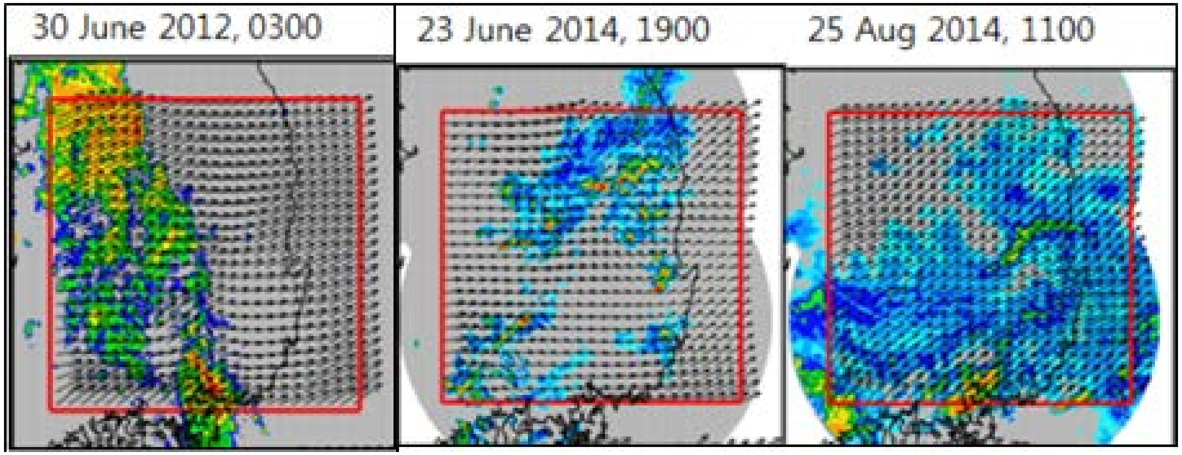


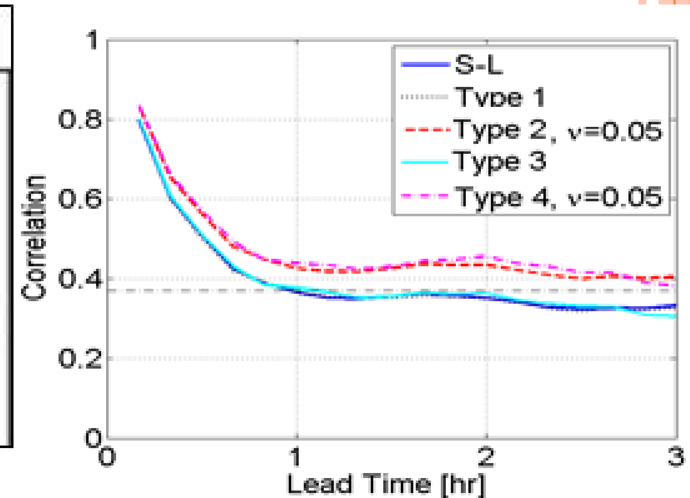


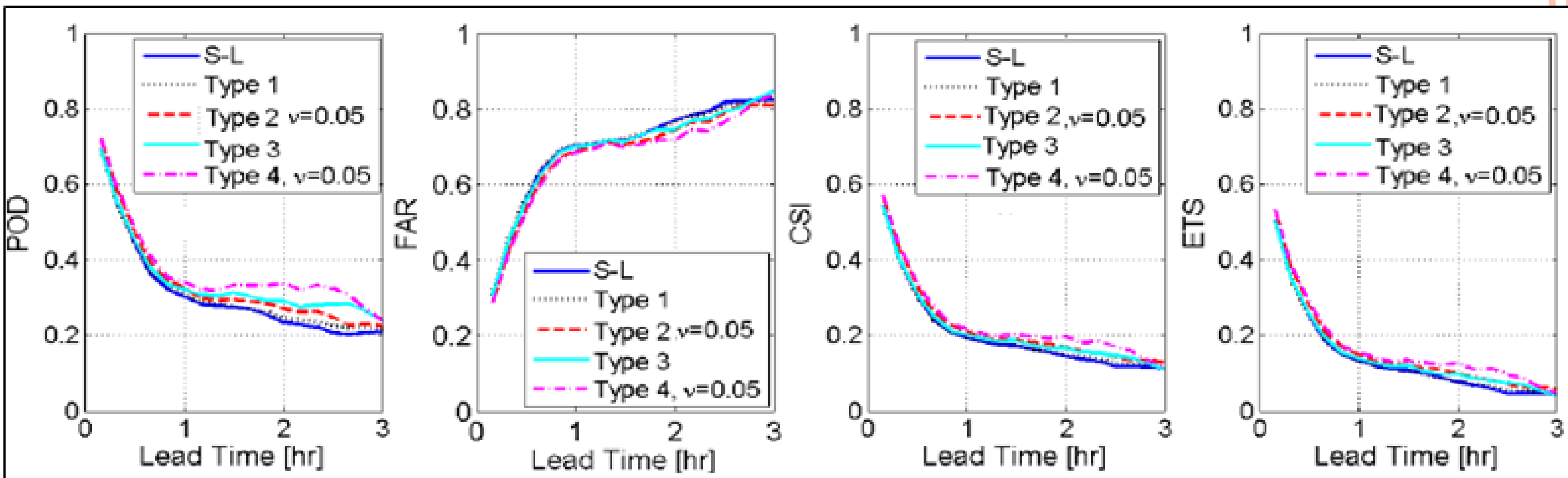
Type 3, 4 (inclusion of Burgers' equation) outperform

NON-STATIONARY MOTION VECTORS

$$\mathrm{Var}(\|\nabla \cdot \boldsymbol{u}\|) \leq 0.13$$







Type 2, 4 (inclusion of diffusion equation) perform better

SUMMARY

- Introduced nowcasting based on advection (diffusion) equation with Burgers' equation
- Performance:
  MAPLE ~ Advection eq. < Advection eq. + Burgers eq.
  (S-L ~ Type1 < Type2 < Type 3 < Type 4)
- Use of diffusion term and non-stationary motion vector improves forecasting skill scores
- When nonstationarity of motion fields is strong, the precipitation forecasts using Burgers' equation (Type3, Type 4) show significant improvement.